

Chapter 1 Introduction

1. Filters

- A filter is a device or system that processes (or reshapes) the input signal according to some specific rules to generate an output signal.



- Linear filter

There is a linear relationship between input and output of the filter

$$x_1 \longrightarrow y_1$$

$$x_2 \longrightarrow y_2$$

$$\text{then } ax_1 + bx_2 \longrightarrow ay_1 + by_2$$

- Nonlinear filter

There is a nonlinearity between input and output of the filter

e.g.

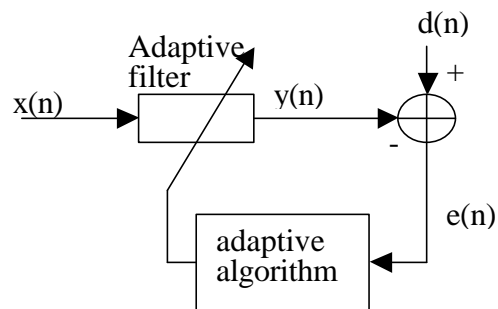
$$x_1 \longrightarrow y_1 = x_1^2$$

$$x_2 \longrightarrow y_2 = x_2^2$$

$$x_1 + x_2 \longrightarrow (x_1 + x_2)^2$$

2. Adaptive Filters

2.1 General Adaptive Filter Configuration



- Characteristics of adaptive filters :

- (1) They can automatically adjust(or adapt) in the face of changing environments and changing system requirements.
- (2) They can be trained to perform specific filtering or decision-making tasks.
- (3) They should be some “adaptation algorithm” for adjusting the system’s parameters.

- Filter Structure

- (1) Transversal filter structure (FIR filter)
- (2) IIR filter structure
- (3) Lattice filter structure
- (4) Nonlinear filter structure (Volterra filter)

- Adaptation Approaches

- (1) Approach based on Wiener filter theory Filter optimization from stochastic framework
The optimum coefficients of a linear filter are obtained by minimization of its mean-square error (MSE).
- (2) Method of least squares Filter optimization from deterministic point of view
The performance index is the sum of weighted error squares.

Adaptive algorithm :

(adaptation algorithm)

- Objective :

To set the filter parameters, $\bar{q}(n)$, in such a way that its output tries to minimize a meaningful objective function F involving the reference signal (or desired signal).

- Minimization algorithms :

- (3) Newton's method
- (4) Quasi-Newton method
- (5) Steepest-descent method

- Definition of objective function $F(n)$:

- (1) mean-square error
- (2) least-square error
- (3) weighted least square
- (4) instantaneous square value

Note :

$$(1) F(e(n)) = E[|e(n)|^2]$$

$$(2) F(e(n)) = \frac{1}{n+1} \sum_{i=0}^n |e(n-i)|^2$$

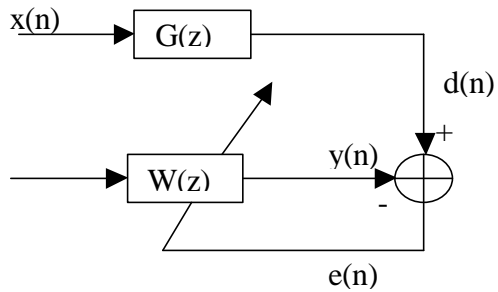
$$(3) F(e(n)) = \sum_{i=0}^n I^i |e(n-i)|^2$$

is a constant smaller than 1.

$$(4) F(e(n)) = |e(n)|^2$$

3. Applications of Adaptive Filters

(a) System modeling



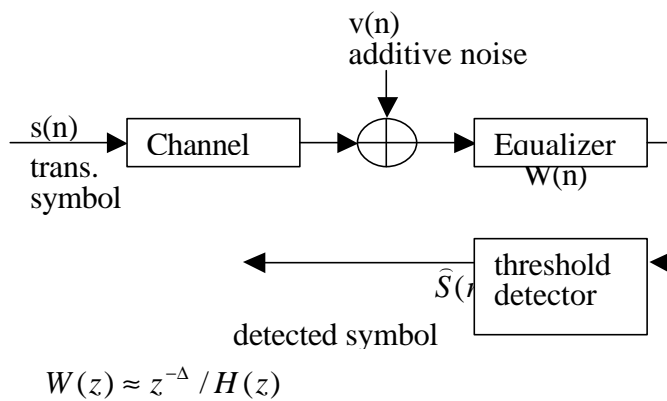
applications :

- system identification
- echo cancellation

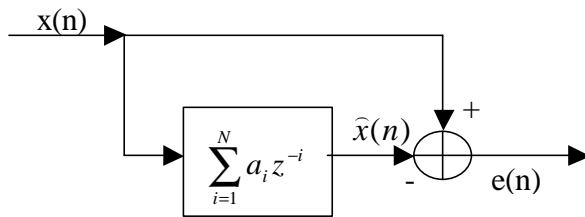
(b) Inverse modeling

Also known as “deconvolution” Application :

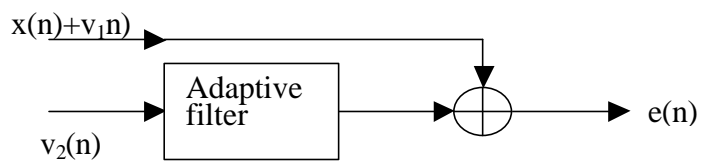
Equalization of communication channel



(c) Signal prediction



(d) Signal enhancement



$v_1(n)$ & $v_2(n)$ are correlated noise signals