Type of Errors

- **Random errors and burst errors.**

Type of Channels

- **Random error channels:**

  Deep space channel, satellite channels, line of sight transmission channel, etc.

- **Burst error channels:**

  Radio links, terrestrial microwave links, wire and cable transmission channels, etc.

Decoding

Suppose a codeword corresponding to a message is transmitted over a noisy channel.

- **Let $\bar{r}$ be the received sequence. Based on $\bar{r}$, the encoding rules and the noise characteristics of the channel, the receiver (or decoder) makes a decision which message was actually transmitted.**

  This decision making operation is called "decoding". The device which performing the decoding operation is called a "decoder".

- **Two types of decoding:**

  Hard-decision decoding and soft-decision decoding.

- **Hard-decision decoding:**

  When binary coding is used, the modulator has only binary inputs. If binary demodulator output quantization is used, the decoder has only binary inputs. In this case, the demodulator is said to make hard decisions.

Decoding based on hard decision made by the demodulator is called "hard decision decoding".

- **Soft-decision decoding:**

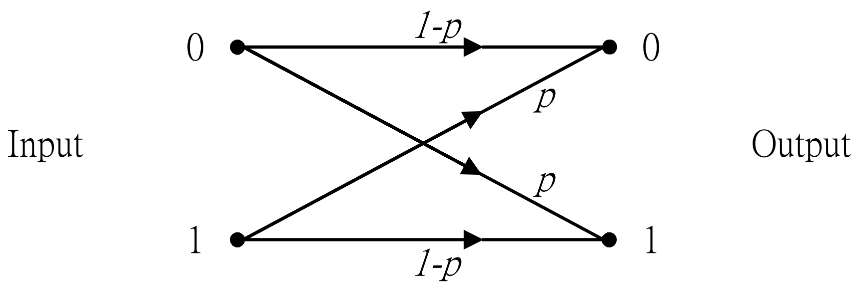
  If the output of demodulator consists of more than two quantization levels or is left unquantized, the demodulator is said to make soft decisions.

  Decoding based on soft decision made by demodulator is called soft-decision decoding.

- **Hard-decision decoding is much easier to implement than soft-decision decoding. However, soft-decision decoding offers much better performance.**

Some Channel Models

- **Binary Symmetric Channel (BSC)**



***p*: transition probability**

When BPSK modulation is used over the AWGN channel with optimum coherent detection and binary output quantization, the bit-error probability for equally likely signal is given by

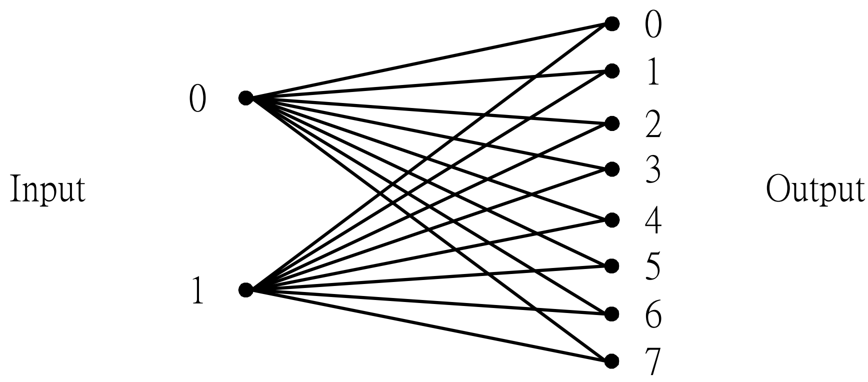
$$p = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

where $Q(x) \equiv \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$

E **: bit energy**

$\frac{N_0}{2}$ **: power spectral density of AWGN**

- **Binary-input, 8-ary Output Discrete Channel**



Optimum Decoding

- **Suppose the codeword $\bar{c}$ corresponding to a certain message $\bar{m}$ is transmitted. Let $\bar{r}$ be the corresponding output of the demodulator.**

- **The decoder produces an estimate $\hat{m}$ of the message based on $\bar{r}$.**

- **An optimum decoding rule is that minimize the probability of a decoding error. That is, $P(\hat{c} \neq \bar{c} \mid \bar{r})$ is minimized. Or, equivalently, maximizing $P(\hat{c} = \bar{c} \mid \bar{r})$.**

- **The decoding error is minimized for a given $\bar{r}$ by choosing $\hat{c}$ to be a codeword $\bar{c}$ that maximizes**

$$P(\bar{c} \mid \bar{r}) = \frac{P(\bar{r} \mid \bar{c})P(\bar{c})}{P(\bar{r})}$$

That is, $\hat{c}$ is chosen to be the most likely codeword, given that $\bar{r}$ is received.

Maximum a posteriori decoding (MAP decoding)

- **In general, we have**

$$P(\bar{c} \mid \bar{r}) = \frac{P(\bar{r} \mid \bar{c})P(\bar{c})}{P(\bar{r})}$$

If knowledge or an estimation of $P(\bar{c})$ is used for decoding, the technique is called MAP decoding.

Maximum Likelihood Decoding (MLD)

- **Suppose all the messages are equally likely. An optimum decoding can be done as follows:**

  For every codeword $\bar{c}_j$, compute the conditional probability

  $$P(\bar{r} \mid \bar{c}_j)$$

  The codeword $\bar{c}_j$ with the largest conditional probability is chosen as the estimate $\hat{c}$ for the transmitted codeword $\bar{c}$.

  This decoding rule is called the Maximum Likelihood decoding (MLD).

- **MLD for a BSC**

  Let $\bar{a} = (a_1, a_2, \cdots, a_n)$ and $\bar{b} = (b_1, b_2, \cdots, b_n)$ be two binary sequences of n components. The Hamming distance between $\bar{a}$ and $\bar{b}$, denoted as $d_H(\bar{a}, \bar{b})$, is defined as the number of places where $\bar{a}$ and $\bar{b}$ differ.

  In coding for a BSC, every codeword and every received sequence are binary sequences.

  Suppose some codeword is transmitted and the received sequence is $\bar{r} = (r_1, r_2, \cdots, r_n)$

For a codeword $\bar{c}_i$, the conditional probability $P(\bar{r}\,|\,\bar{c}_i)$ is

$$P(\bar{r}\,|\,\bar{c}_i) = p^{d_H(\bar{r},\bar{c}_i)}(1-p)^{n-d_H(\bar{r},\bar{c}_i)}$$

For $p < \frac{1}{2}$, $P(\bar{r}\,|\,\bar{c}_i)$ is a monotonically decreasing function of $d_H(\bar{r},\bar{c}_i)$.

Then $P(\bar{r}\,|\,\bar{c}_i) > P(\bar{r}\,|\,\bar{c}_j)$ if and only if

$$d_H(\bar{r},\bar{c}_i) < d_H(\bar{r},\bar{c}_j)$$

The MLD is completed by the following steps:

(i) Compute $d_H(\bar{r},\bar{c}_i)$ for all $\bar{c}_i$

(ii) $\bar{c}_i$ is taken as the transmitted codeword if $d_H(\bar{r},\bar{c}_i) < d_H(\bar{r},\bar{c}_j)$ for $j \neq i$

(iii) Decode $\bar{c}_i$ into message $\bar{m}_i$

That is, the received vector $\bar{r}$ is decoded into the closest codeword.

This is also called the minimum distance (nearest neighbor) decoding.

Bounded Distance Decoding

- **Given a received word $\bar{r}$, a t-error correcting, bounded distance decoder selects that codeword $\bar{c}$ which minimizes $d_H(\bar{r},\bar{c})$ if and only if there exists $\bar{c}$ such that $d_H(\bar{r},\bar{c}) \leq t$. If no such $\bar{c}$ exists, then a decoder failure is declared.**

✶ **All practical bounded distance decoder use some form of syndrome decoding.**

✶ **The bounded distance decoding is usually an "incomplete decoding" since it decodes only those received words lying in a radius-t sphere about a codeword.**